Matematika (bahasa Yunani: mathēmatikā, "pengetahuan/belajar") adalah ilmu tentang besaran, struktur, ruang, dan perubahan.

LIMIT FUNGSI

19

A. PENGERTIAN LIMIT FUNGSI

a. Pengertian Limit

Limit suatu fungsi f(x) untuk x mendekati nilai a adalah harga yang paling dekat dari f(x) pada saat x mendekati nilai a, dari kiri dan dari kanan.

$\lim\limits_{x \to a} f(x)$ terdefinisi jika dan hanya jika $\lim\limits_{x \to a_-} f(x) = \lim\limits_{x \to a_+}$ atau **limit kiri** sama dengan **limit kanan**.

Jadi, jika $\lim\limits_{x \to a} f(x) = fA$ artinya **fA** adalah nilai pendekatan untuk x di sekitar a.

Untuk c konstanta, dalam konteks limit berlaku: $\frac{c}{0} = \infty, \frac{0}{c} = 0, \frac{c}{\infty} = 0, \frac{\infty}{c} = \infty.$

b. Teorema Limit

- Jika f(x) = x, maka $\lim\limits_{x \to a} f(x) = a$

- $\lim_{x \to a} \left(f(x) \pm g(x) \right) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
- Jika k konstanta, maka $\lim_{x \to a} k.f(x) = k.\lim_{x \to a} f(x)$
- $\lim_{x \to a} \left(f(x).g(x) \right) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \lim_{x \to a} g(x) \neq 0$
- $\lim_{x \to a} \{f(x)\}^n = \left\{ \lim_{x \to a} f(x) \right\}^n$

B. LIMIT FUNGSI ALJABAR

a. Langkab Umum Penyelesaian

$\text{Limit}_{x \to a} f(x) = ...$

- Substitusikan $x = a$ ke $f(x)$
- Jika hasilnya bentuk tak tentu $\left(\frac{0}{0}, \frac{\infty}{\infty}, \text{dan}\, \infty - \infty \right)$, maka $f(x)$ harus diuraikan
- Jika hasilnya bentuk tertentu, maka itulah nilai limitnya.

b. Cara Menguraikan Fungsi f(x)

1. Untuk $\mathbf{x \to c}$, c (konstanta) dan hasilnya $\left(\frac{0}{0} \right)$, maka fungsi $f(x)$ diuraikan dengan cara:

- Faktorisasi
- Kali sekawan, jika f(x) mengandung bentuk akar ($\sqrt{\ }$)
- Dalil L'Hospital $\rightarrow \underset{x \to c}{\text{Limit}} \dfrac{f(x)}{g(x)} = \underset{x \to c}{\text{Limit}} \dfrac{f'(x)}{g'(x)}$s

2. Untuk $x \to \infty$ dan hasilnya $\left(\dfrac{\infty}{\infty}\right)$, maka fungsi f(x) diuraikan dengan cara:

   - Membagi pembilang dan penyebut dengan x pangkat tertinggi.
   - Gunakan rumus: $\underset{x \to \infty}{\text{Limit}} \dfrac{a_1x^m + a_2x^{m-1} + ... + a}{b_1x^n + b_2x^{n-1} + ... + b} = \begin{cases} \infty, \text{untuk } m > n \\ \frac{a_1}{b_1}, \text{untuk } m = n \\ 0, \text{untuk } m < n \end{cases}$

3. Untuk $x \to \infty$ dan hasilnya $(\infty - \infty)$, maka fungsi f(x) diuraikan dengan cara:

   - Kali sekawan jika f(x) mengandung bentuk akar kemudian membagi pembilang dan penyebut dengan x pangkat tertinggi.
   - Gunakan rumus selisih akar kuadrat:

$$\underset{x \to \infty}{\text{Limit}} \left(\sqrt{ax^2 + bx + c} - \sqrt{px^2 + qx + r}\right) = \begin{cases} \infty, \quad \text{untuk } a > p \\ \frac{b-q}{2\sqrt{a}}, \text{untuk } a = p \\ -\infty, \text{untuk } a < p \end{cases}$$

CONTOH SOAL & PEMBAHASAN - 1

Untuk $x \to c$

1. Nilai $\lim\limits_{x \to 2} \dfrac{x^2 + 8x - 20}{x^2 - 5x + 6} = ...$

   A. 12
   B. 10
   C. 2
   D. −10
   E. −12

Jawaban: E

$$\Leftrightarrow \lim_{x\to 2}\frac{x^2+8x-20}{x^2-5x+6}=\lim_{x\to 2}\frac{(x+10)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}$$

$$=\frac{(2+10)}{(2-3)}=-12$$

SOLUSI SMART!

Gunakan dalil L'HOSPITAL

$$\underset{x\to c}{\text{Limit}}\ \frac{f(x)}{g(x)}=\underset{x\to c}{\text{Limit}}\ \frac{f'(x)}{g'(x)}$$

$$\Leftrightarrow \lim_{x\to 2}\frac{x^2+8x-20}{x^2-5x+6}$$

$$=\lim_{x\to 2}\frac{2x+8}{2x-5}=\frac{2(2)+8}{2(2)-5}=-12$$

2. $\lim\limits_{x\to 1}\dfrac{(x-1)(\sqrt{x}+1)}{\sqrt{x}-1}=\ldots$

A. 0
B. 1
C. 2
D. 4
E. 8

Jawaban: D

$$\Leftrightarrow \lim_{x\to 1}\frac{(x-1)(\sqrt{x}+1)}{\sqrt{x}-1}$$

$$=\lim_{x\to 1}\frac{\overbrace{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}^{(x-1)}(\sqrt{x}+1)}{\cancel{\sqrt{x}-1}}$$

$$=(\sqrt{1}+1)(\sqrt{1}+1)=4$$

3. $\lim\limits_{x \to 1} \left(\frac{1}{2x-2} - \frac{1}{x^2-1} \right) = ...$

A. $-\frac{1}{2}$

B. $-\frac{1}{4}$

C. 0

D. $\frac{1}{4}$

E. $\frac{1}{2}$

Jawaban: D

$$\Leftrightarrow \lim_{x \to 1} \left(\frac{1}{2(x-1)} - \frac{1}{(x-1)(x+1)} \right)$$

$$= \lim_{x \to 1} \left(\frac{(x+1)-2}{2(x-1)(x+1)} \right)$$

$$= \lim_{x \to 1} \left(\frac{(x-\cancel{1})}{2\cancel{(x-1)}(x+1)} \right)$$

$$= \frac{1}{2(1+1)} = \frac{1}{4}$$

4. $\lim\limits_{x \to a} \frac{x^2+(3-a)x-3a}{x-a} = ...$

A. a

B. a + 1

C. a + 2

D. a + 3

E. a + 4

Jawaban: D

$$\Leftrightarrow \lim_{x \to a} \frac{x^2+(3-a)x-3a}{x-a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x+3)}{\cancel{(x-a)}}$$

$$= (a+3)$$

SOLUSI SMART!

Gunakan dalil L 'HOSPITAL

$\Leftrightarrow \lim\limits_{x \to a} \dfrac{x^2 + (3-a)x - 3a}{x-a}$

$= \lim\limits_{x \to a} \dfrac{2x + (3-a)}{1}$

$= 2a + 3 - a = a + 3$

5. Nilai $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{4 - \sqrt{5x+1}} = ...$

A. -8
B. -6
C. 6
D. 8
E. ∞

Jawaban: A

SOLUSI SMART!

- **Gunakan dalil L 'HOSPITAL**
- $y = \sqrt{u} \Rightarrow y' = \dfrac{u'}{2\sqrt{u}}$

$\Leftrightarrow \lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{4 - \sqrt{5x+1}} = \lim\limits_{x \to 3} \dfrac{2x - 1}{-\left(\dfrac{5}{2\sqrt{5x+1}}\right)}$

$= \dfrac{2(3) - 1}{-\left(\frac{5}{2\sqrt{16}}\right)} = -8$

6. $\lim\limits_{x \to 0} \dfrac{f(a-x)-f(a)}{x} = ...$

A. f'A.
B. –f'A.
C. f'(x)
D. –f'(x)
E. f A.

Jawaban: B

SOLUSI SMART!

Gunakan dalil L'HOSPITAL

$$\Leftrightarrow \lim\limits_{x \to 0} \frac{f(a-x)-f(a)}{x}$$

$$= \lim\limits_{x \to 0} \frac{-f'(a-x)-0}{1} = -f'(a)$$

7. Diketahui fungsi g kontinu di x = 3 dan $\lim\limits_{x \to 3} g(x) = 2$. Nilai $\lim\limits_{x \to 3} \left(g(x) \dfrac{x-3}{\sqrt{x}-\sqrt{3}} \right)$ adalah ....

A. $4\sqrt{3}$
B. $2\sqrt{3}$
C. $\sqrt{3}$
D. 4
E. 2

Jawaban: A

$$\Leftrightarrow \lim\limits_{x \to 3} \left(g(x) \frac{x-3}{\sqrt{x}-\sqrt{3}} \right)$$

$$= \lim\limits_{x \to 3} \left((2) \frac{(\cancel{\sqrt{x}-\sqrt{3}})(\sqrt{x}+\sqrt{3})}{(\cancel{\sqrt{x}-\sqrt{3}})} \right)$$

$$= (2)(\sqrt{3}+\sqrt{3}) = 4\sqrt{3}$$

8. Jika $\lim\limits_{x \to 4} \dfrac{ax + b - \sqrt{x}}{x - 4} = \dfrac{3}{4}$, maka a + b sama dengan ....

A. 3
B. 2
C. 1
D. −1
E. −2

Jawaban: D

⇔ Untuk $x = 4$, $\dfrac{4a + b - \sqrt{4}}{4 - 4} = \dfrac{0}{0}$

Gunakan dalil L 'HOSPITAL

⇔ $\lim\limits_{x \to 4} \dfrac{a - \frac{1}{2\sqrt{x}}}{1} \to a = -\dfrac{1}{2\sqrt{4}} = \dfrac{3}{4} \to a = 1$

⇔ Untuk a =1 substitusikan ke pers (1), diperoleh:
$4 + b - 2 = 0 \to b = -2$

⇔ Jadi, $a + b = -1$

9. $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{\sqrt[3]{1 + x} - 1} = ...$

A. 0
B. $\frac{2}{3}$
C. 1
D. $\frac{3}{2}$
E. ∞

Jawaban: D

⇔ Misal : $p = (1 + x)^{\frac{1}{6}}$

$$\sqrt{1 + x} = (1 + x)^{\frac{3}{6}} = p^3$$

$$\sqrt[3]{1 + x} = (1 + x)^{\frac{2}{6}} = p^2$$

- untuk $x \to 0$, maka $p \to 1$

⇔ $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x} - 1}{\sqrt[3]{1 + x} - 1}$

$= \lim\limits_{p \to 1} \dfrac{p^3 - 1}{p^2 - 1}$

$$= \lim_{p \to 1} \frac{\cancel{(p-1)}(p^2+p+1)}{\cancel{(p-1)}(p+1)}$$

$$= \frac{(1+1+1)}{(1+1)} = \frac{3}{2}$$

SOLUSI SMART!

Gunakan dalil L 'HOSPITAL

$$\Leftrightarrow \lim_{x \to 0} \frac{\frac{1}{2\sqrt{1+x}}}{\frac{1}{3}(1+x)^{-\frac{2}{3}}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$$

10. Jika $\lim\limits_{x \to 0} \frac{g(x)}{x} = 2$ maka $\lim\limits_{x \to 0} \frac{g(x)}{\sqrt{1-x}-1} = ...$

A. -4

B. -2

C. 1

D. 2

E. 4

Jawaban: A

SOLUSI SMART!

Gunakan dalil L 'HOSPITAL

$$\lim_{x \to 0} \frac{g(x)}{x} = \lim_{x \to 0} \frac{g'(x)}{1} \to g'(0) = 2$$

$$\Leftrightarrow \lim_{x \to 0} \frac{g(x)}{\sqrt{1-x}-1} = \lim_{x \to 0} \frac{g'(x)}{\frac{-1}{2\sqrt{1-x}}}$$

$$= \frac{g'(0)}{\frac{-1}{2\sqrt{1}}}$$

$$= \frac{2}{\frac{-1}{2\sqrt{1}}} = -4$$

11. Jika $\lim\limits_{x \to 0} \dfrac{\sqrt{a+x}-\sqrt{a-x}}{x} = b$, maka $\lim\limits_{x \to 0} \dfrac{\sqrt{b+x}-\sqrt{b-x}}{x}$ sama dengan ....

A. a
B. $\dfrac{1}{\sqrt{a}}$
C. $\dfrac{1}{\sqrt{\sqrt{a}}}$
D. $\sqrt{a}$
E. $\sqrt{\sqrt{a}}$

Jawaban: E

Gunakan dalil L 'HOSPITAL

$$\Leftrightarrow \lim\limits_{x \to 0} \frac{\dfrac{1}{2\sqrt{a+x}} - \left(\dfrac{-1}{2\sqrt{a-x}}\right)}{1} = b$$

$$\frac{1}{2\sqrt{a}} + \frac{1}{2\sqrt{a}} = b \rightarrow \frac{1}{\sqrt{a}} = b$$

$$\Leftrightarrow \lim\limits_{x \to 0} \frac{\sqrt{b+x}-\sqrt{b-x}}{x} = \frac{1}{\sqrt{b}}$$

Caranya sama dengan di atas

$$= \frac{1}{\sqrt{\dfrac{1}{\sqrt{a}}}} = \sqrt{\sqrt{a}}$$

Untuk $x \to \infty$

12. Nilai $\lim\limits_{x \to \infty} \dfrac{5x^2 - 22x + 21}{4x^2 + 2x - 15} = ...$

A. 5
B. $\frac{5}{4}$
C. 0
D. $-\frac{5}{4}$
E. -5

Jawaban: B

$$\Leftrightarrow \lim_{x\to\infty} \frac{5x^2-22x+21}{4x^2+2x-15}$$
$$= \lim_{x\to\infty} \frac{\cancel{x^2}(5-\frac{22}{x}+\frac{21}{x^2})}{\cancel{x^2}(4+\frac{2}{x}-\frac{15}{x^2})}$$
$$= \frac{(5-0+0)}{(4+0-0)} = \frac{5}{4}$$

SOLUSI SMART!

- $$\lim_{x\to\infty} \frac{a_1x^m + a_2x^{m-1}+\dots}{b_1x^n + b_2x^{n-1}+\dots} = \begin{cases} m>n, \text{ hasilnya} = \infty \\ m=n, \text{ hasilnya} = \frac{a_1}{b_1} \\ m<n, \text{ hasilnya} = 0 \end{cases}$$

$$\Leftrightarrow \lim_{x\to\infty} \frac{5x^2-22x+21}{4x^2+2x-15}$$

- $m = n = 2 \rightarrow \lim_{x\to\infty} \frac{5x^2-22x+21}{4x^2+2x-15} = \frac{5}{4}$

13. Nilai $\lim_{x\to\infty} \frac{2x^2+3x}{\sqrt{x^2-x}} = \dots$

A. 0
B. $\frac{1}{2}$
C. 1
D. 2
E. ∞

Jawaban: E

$$\Leftrightarrow \lim_{x\to\infty} \frac{2x^2+3x}{\sqrt{x^2-x}}$$
$$= \lim_{x\to\infty} \frac{x(2x+3)}{x\left(\sqrt{1-\frac{1}{x}}\right)} = \frac{\infty}{\sqrt{1-0}} = \infty$$

SOLUSI SMART!

- $$\underset{x\to\infty}{\text{Limit}} \frac{a_1x^m + a_2x^{m-1} + ...}{b_1x^n + b_2x^{n-1} + ...} = \begin{cases} m > n, \text{ hasilnya} = \infty \\ m = n, \text{ hasilnya} = \frac{a_1}{b_1} \\ m < n, \text{ hasilnya} = 0 \end{cases}$$

$$\Leftrightarrow \lim_{x\to\infty} \frac{2x^2+3x}{\sqrt{x^2-x}} = \infty \to m > n$$

14. $\underset{x\to\infty}{\text{Limit}} \frac{(1-2x)^3}{(x-1)(2x^2+x+1)} = ...$

A. -8
B. -4
C. $\frac{1}{2}$
D. 4
E. 8

Jawaban: B

$$\Leftrightarrow \underset{x\to\infty}{\text{Limit}} \frac{(1-2x)^3}{(x-1)(2x^2+x+1)}$$

pembilang dan penyebut dibagi x^3

$$= \underset{x\to\infty}{\text{Limit}} \frac{\left(\frac{1-2x}{x}\right)^3}{\left(\frac{x-1}{x}\right)\left(\frac{2x^2+x+1}{x^2}\right)}$$

$$= \frac{(0-2)^3}{(1-0)(2+0+0)} = \frac{-8}{2} = -4$$

SOLUSI SMART!

lihat koefisien pangkat tertinggi x^3 pada pembilang dan penyebut

$$\Leftrightarrow \underset{x\to\infty}{\text{Limit}} \frac{(1-2x)^3}{(x-1)(2x^2+x+1)} = \frac{(-2)^2}{2} = -4$$

15. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$ sama dengan ....

A. 2

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{3}$

E. 0

Jawaban: B

$$\Leftrightarrow \lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \left(\frac{\sqrt{x}}{\sqrt{x}}\right)$$

$$= \lim_{x \to \infty} \frac{x}{\sqrt{x^2 + x\sqrt{x+\sqrt{x}}}}$$

pembilang dan penyebut di bagi x

$$= \lim_{x \to \infty} \frac{x}{x\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}$$

$$= \lim_{x \to \infty} \frac{1}{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^2}}}}$$

$$= \frac{1}{1+\sqrt{0+\sqrt{0}}}$$

SOLUSI SMART!

lihat koefisien pangkat tertinggi $(\sqrt{x})$ pada pembilang dan penyebut

$$\Leftrightarrow \lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \frac{1}{1}$$

16. $\lim\limits_{x \to \infty} \dfrac{2^{x+1} - 3^{x-2} + 4^{x+1}}{2^{x-1} + 3^{x+1} + 4^{x-1}} = \ldots$

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. 16

E. 32

Jawaban: D

$$\Leftrightarrow \lim_{x \to \infty} \frac{2^{x+1} - 3^{x-2} + 4^{x+1}}{2^{x-1} + 3^{x+1} + 4^{x-1}}$$

pembilang & penyebut dibagi (4^x),

$$= \lim_{x \to \infty} \frac{\dfrac{2^{x+1} - 3^{x-2} + 4^{x+1}}{4^x}}{\dfrac{2^{x-1} + 3^{x+1} + 4^{x-1}}{4^x}}$$

$$= \lim_{x \to \infty} \frac{2.2^{-x} - 3^{-2}.\left(\frac{3}{4}\right)^x + 4}{2^{-1}.2^{-x} + 3.\left(\frac{3}{4}\right)^x + 4^{-1}.}$$

$$= \frac{0 - 0 + 4}{0 + 0 + \frac{1}{4}} = 16$$

17. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 2x - 3} - \sqrt{x^2 - 2x + 14} \right) = \ldots$

A. ∞

B. 5

C. 4

D. 2

E. 0

Jawaban: D

$$\Leftrightarrow \lim_{x \to \infty} \left(\sqrt{\underbrace{x^2 + 2x - 3}_{a}} - \sqrt{\underbrace{x^2 - 2x + 14}_{b}} \right)$$

$$= \lim_{x \to \infty} \left(\sqrt{a} - \sqrt{b} \right) \left(\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} \right)$$

$$= \lim_{x \to \infty} \left(\frac{a-b}{\sqrt{a}+\sqrt{b}} \right)$$

$$= \lim_{x \to \infty} \left(\frac{(x^2+2x-3)-(x^2-2x+14)}{\sqrt{x^2+2x-3}+\sqrt{x^2-2x+14}} \right)$$

$$= \lim_{x \to \infty} \frac{4x}{\sqrt{x^2\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}+\sqrt{x^2\left(1-\frac{2}{x}+\frac{14}{x^2}\right)}}$$

$$= \lim_{x \to \infty} \frac{4x}{x\left(\sqrt{\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}+\sqrt{\left(1-\frac{2}{x}+\frac{14}{x^2}\right)}\right)}$$

pembilang dan penyebut dibagi x

$$= \lim_{x \to \infty} \frac{4}{\sqrt{\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}+\sqrt{\left(1-\frac{2}{x}+\frac{14}{x^2}\right)}}$$

$$= \lim_{x \to \infty} \frac{4}{\sqrt{(1+0-0)}+\sqrt{(1-0+0)}}$$

$$= \frac{4}{2} = 2$$

SOLUSI SMART!

Gunakan rumus selisih akar kuadrat:

$$\underset{x \to \infty}{\text{Limit}} \left(\sqrt{ax^2+bx+c} - \sqrt{ax^2+qx+r} \right) = \frac{b-q}{2\sqrt{a}}$$

$$\Leftrightarrow \lim_{x \to \infty} \left(\sqrt{x^2+2x-3} - \sqrt{x^2-2x+14} \right)$$

- $a = 1, b = 2$ dan $q = -2$

$$= \lim_{x \to \infty} \left(\sqrt{x^2+2x-3} - \sqrt{x^2-2x+14} \right)$$

$$= \frac{b-q}{2\sqrt{a}} = \frac{2-(-2)}{2\sqrt{1}} = \frac{4}{2} = 2$$

18. Nilai dari $\lim\limits_{x \to \infty} \left(\sqrt{x(4x+5)} - 2x + 1 \right) = ...$

A. 0
B. $\frac{1}{4}$
C. $\frac{1}{2}$
D. $\frac{9}{4}$
E. ∞

Jawaban: D

SOLUSI SMART!

Gunakan rumus selisih akar kuadrat:

$\Leftrightarrow \lim\limits_{x \to \infty} \left(\sqrt{x(4x+5)} - 2x + 1 \right)$

$= \lim\limits_{x \to \infty} \left(\sqrt{4x^2 + 5x} - \sqrt{(2x-1)^2} \right)$

$= \lim\limits_{x \to \infty} \left(\sqrt{4x^2 + 5x} - \sqrt{4x^2 - 4x + 1} \right)$

$= \frac{b-q}{2\sqrt{a}} = \frac{5-(-4)}{2\sqrt{4}} = \frac{9}{4}$

19. $\underset{x \to \infty}{\text{Limit}} \left(\sqrt{x + \sqrt{2x}} - \sqrt{x} \right) = ...$

A. $\sqrt{2\sqrt{2}}$
B. 2
C. $\sqrt{2}$
D. $\frac{1}{2}\sqrt{2}$
E. 0

Jawaban: D

SOLUSI SMART!

Gunakan rumus selisih akar kuadrat:

$\Leftrightarrow$ Misal: $y = \sqrt{x}$, maka limit di atas menjadi:

$\underset{x \to \infty}{\text{Limit}} \left(\sqrt{y^2 + \sqrt{2}\,y} - \sqrt{y^2} \right)$

$$\rightarrow a = 1,\ b = \sqrt{2}\ ,\ \text{dan } q = 0$$

$$= \frac{b-q}{2\sqrt{a}} = \frac{\sqrt{2}-0}{2.1} = \tfrac{1}{2}\sqrt{2}$$

20. $\underset{x \to \infty}{\text{Limit}} \left(\sqrt{(x+a)(x+b)} - x\right) = ...$

A. $\frac{a-b}{2}$

B. ∞

C. 0

D. $\frac{a+b}{2}$

E. $a + b$

Jawaban: D

$\Leftrightarrow \underset{x \to \infty}{\text{Limit}} \left(\sqrt{(x+a)(x+b)} - x\right)$

Gunakan rumus selisih akar kuadrat:

$= \underset{x \to \infty}{\text{Limit}} \left(\sqrt{x^2 + (a+b)x + ab} - \sqrt{x^2}\right)$

$= \frac{(a+b)-0}{2\sqrt{1}} = \frac{a+b}{2}$

21. Nilai $\underset{x \to \infty}{\lim} \left(\sqrt{4x^2 + 8x} - \sqrt{x^2+1} - \sqrt{x^2+x}\right) = ...$

A. $\frac{5}{2}$

B. 2

C. $\frac{3}{2}$

D. 1

E. $\frac{1}{2}$

Jawaban: C

SOLUSI SMART!

$\underset{x \to \infty}{\lim} \left(\sqrt{ax^2+bx+c} + \sqrt{dx^2+ex+f} + \sqrt{gx^2+hx+i} + ...\sqrt{...}\right)$

$= \left(\frac{b}{2\sqrt{a}}\right) + \left(\frac{e}{2\sqrt{d}}\right) + \left(\frac{h}{2\sqrt{g}}\right) + ...$

Rumus ini hanya berlaku jika

$\sqrt{a}+\sqrt{d}+\sqrt{g}+\ldots=\mathbf{0}$

$$\Leftrightarrow \lim_{x\to\infty}\left(\sqrt{4x^2+8x}-\sqrt{x^2+1}-\sqrt{x^2+x}\right)$$

$$=\left(\frac{8}{2\sqrt{4}}\right)-\left(\frac{0}{2\sqrt{1}}\right)-\left(\frac{1}{2\sqrt{1}}\right)$$

$$=2-\tfrac{1}{2}=\tfrac{3}{2}$$

C. LIMIT FUNGSI TRIGONOMETRI

a. Langkah Umum Penyelesaian

$$\text{Limit}_{x\to a}\ f(x)=\ldots\ldots$$

- Substitusikan $x=a$ ke $f(x)$
- Jika hasilnya bentuk tak tentu $\left(\frac{0}{0},\frac{\infty}{\infty},(\infty-\infty),\text{ dan }(0.\infty)\right)$, maka

  f(x) harus diuraikan
- Jika hasilnya bentuk tertentu, maka itulah nilai limitnya.

b. Cara Menguraikan Fungsi f(x)

1. Untuk bentuk dasar, gunakan rumus:

- $\text{Limit}_{x\to 0}\ \frac{\sin ax}{bx}=\frac{a}{b}$
- $\text{Limit}_{x\to 0}\ \frac{ax}{\tan bx}=\frac{a}{b}$
- $\text{Limit}_{x\to 0}\ \frac{ax}{\sin\ bx}=\frac{a}{b}$
- $\text{Limit}_{x\to 0}\ \frac{\tan ax}{\sin bx}=\frac{a}{b}$
- $\text{Limit}_{x\to 0}\ \frac{\tan ax}{bx}=\frac{a}{b}$
- $\text{Limit}_{x\to 0}\ \frac{\sin ax}{\tan bx}=\frac{a}{b}$

2. Jika terdapat bentuk $\lim\limits_{x \to c} \frac{f(x)}{g(x)}$ dan hasilnya $\left(\frac{0}{0}\right)$ dimana f(x) dan g(x) mudah diturunkan gunakan cara **dalil L ' Hospital.**

3. Jika terdapat bentuk **identitas trigonometri** gunakan rumus:

- $1 - \cos cx = 2\sin^2 \frac{c}{2}x$
- $1 + \cos cx = 2\cos^2 \frac{c}{2}x$
- $\sin cx = 2\sin \frac{c}{2}x \cos \frac{c}{2}x$
- $1 - \sin 2cx = \cos^2 cx$
- $1 - \cos 2cx = \sin^2 cx$

CONTOH SOAL & PEMBAHASAN - 2

22. Nilai $\lim\limits_{x \to 0} \frac{2.\tan 2x}{3x} = ...$

A. ∞
B. 0
C. $\frac{2}{3}$
D. $\frac{4}{3}$
E. $\frac{8}{3}$

Jawaban: D

$$\Leftrightarrow \lim_{x \to 0} 2.\left(\frac{\tan 2x}{3x}\right) = 2.\left(\frac{2}{3}\right) = \frac{4}{3}$$

23. $\lim\limits_{x \to 0} \frac{\sin 2x.\tan 3x}{x.\sin x} = ...$

A. 0
B. $\frac{1}{6}$
C. 5
D. 6
E. ∞

Jawaban: D

$$\Leftrightarrow \lim_{x \to 0} \frac{\sin 2x.\tan 3x}{x.\sin x}$$

$$= \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)\left(\frac{\tan 3x}{\sin x}\right) = \left(\frac{2}{1}\right)\left(\frac{3}{1}\right) = 6$$

24. Nilai $\underset{x \to 0}{\text{Limit}} \dfrac{\cos 4x.\, \sin 3x}{5x} = ...$

A. 0
B. 0,2
C. 0,6
D. 1
E. ∞

Jawaban: C

$$\Leftrightarrow \underset{x \to 0}{\text{Limit}} \left\{ \left(\frac{\cos 4x}{1} \right) \left(\frac{\sin 3x}{5x} \right) \right\} = \left(\frac{1}{1} \right) \left(\frac{3}{5} \right) = 0,6$$

25. $\underset{x \to 0}{\text{Limit}} \dfrac{x^2 + tg^2 3x}{\sin(2x^2)} = ...$

A. 2
B. $2\frac{1}{2}$
C. -2
D. 5
E. -5

Jawaban: D

$$\Leftrightarrow \underset{x \to 0}{\text{Limit}} \frac{x^2 + tg^2 3x}{\sin(2x^2)}$$

$$= \underset{x \to 0}{\text{Limit}} \left(\frac{x^2}{\sin 2x^2} + \frac{tg^2 3x}{\sin 2x^2} \right)$$

$$= \left(\frac{1}{2} + \frac{9}{2} \right) = 5$$

26. $\underset{x \to \pi}{\lim} \dfrac{x - \pi}{2(x - \pi) + \tan(x - \pi)} = ...$

A. $-\frac{1}{2}$
B. $-\frac{1}{4}$
C. $\frac{1}{4}$
D. $\frac{1}{3}$
E. $\frac{2}{5}$

Jawaban: D

$$\Leftrightarrow \lim_{x \to \pi} \frac{(x-\pi)}{2(x-\pi)+\tan(x-\pi)}$$

- **pembilangan dan penyebut dibagi (x - π)**

$$= \lim_{x \to \pi} \frac{1}{2+\frac{\tan(x-\pi)}{(x-\pi)}} = \frac{1}{2+1} = \frac{1}{3}$$

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$$\Leftrightarrow \lim_{x \to \pi} \frac{1}{2+\sec^2(x-\pi)} = \frac{1}{2+1} = \frac{1}{3}$$

27. $\lim_{x \to \frac{\pi}{2}} \frac{\pi(\pi-2x)\tan(x-\frac{\pi}{2})}{2(x-\pi)\cos^2 x} = ...$

A. -2
B. -1
C. $\frac{1}{2}$
D. 1
E. 2

Jawaban: E

$$\Leftrightarrow \lim_{x \to \frac{\pi}{2}} \frac{\pi(\pi-2x)\tan(x-\frac{\pi}{2})}{2(x-\pi)\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \left(\frac{2\pi}{2(x-\pi)}\right)\left(\frac{(\frac{\pi}{2}-x)\tan-(\frac{\pi}{2}-x)}{\sin^2(\frac{\pi}{2}-x)}\right)$$

$$= \left(\frac{2\pi}{2(-\frac{\pi}{2})}\right)\left(\frac{(1)(-1)}{(1)^2}\right)$$

$$= (-2).(-1) = 2$$

28. $\lim\limits_{x \to -1} \dfrac{\sin(1-x^2).\cos(1-x^2)}{x^2-1} = ...$

A. 1
B. −1
C. 2
D. −2
E. 0

Jawaban: B

$$\Leftrightarrow \lim_{x \to -1} \frac{\sin(1-x^2).\cos(1-x^2)}{x^2-1}$$

$$= \lim_{x \to -1} \underbrace{\left(\frac{\sin(1-x^2)}{-(1-x^2)}\right)}_{-1} \underbrace{\cos(1-x^2)}_{1}$$

$$= -1$$

29. Nilai $\lim\limits_{x \to 0} \dfrac{\sin 4x}{1-\sqrt{1-x}} = ...$

A. 8
B. 6
C. 4
D. −6
E. −8

Jawaban: A

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$$\Leftrightarrow \lim_{x \to 0} \frac{\sin 4x}{1-\sqrt{1-x}}$$

$$= \lim_{x \to 0} \frac{4.\cos 4x}{\left(0 - \frac{-1}{2\sqrt{1-x}}\right)} = \frac{4}{\frac{1}{2}} = 8$$

30. $\underset{x \to y}{\text{limit}} \dfrac{\tan x - \tan y}{\left(1-\frac{x}{y}\right).(1+\tan x \cdot \tan y)} = .$

A. -1

B. 1

C. 0

D. y

E. $-y$

Jawaban: E

Catatan : $\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x.\tan y}$

$$\Leftrightarrow \underset{x \to y}{\text{limit}} \left\{ \frac{1}{\left(\frac{y-x}{y}\right)} \right\} \left(\frac{tgx - tgy}{1+tgx.tgy} \right)$$

$$= \underset{x \to y}{\text{limit}} \left(\frac{y}{1} \right) \left(\frac{tg\,(x-y)}{-(x-y)} \right)$$

$$= (y)(-1) = -y$$

31. Nilai $\underset{x \to 0}{\lim} \dfrac{5x.\tan 3x}{1-\cos 6x} = ...$

A. 0

B. $\frac{5}{9}$

C. $\frac{5}{6}$

D. $\frac{2}{3}$

E. ∞

Jawaban: C

$$\Leftrightarrow \underset{x \to 0}{\lim} \frac{5x.\tan 3x}{1-\cos 6x}$$

$$= \underset{x \to 0}{\lim} \frac{5x.\tan 3x}{2\sin^2 3x}$$

$$= \underset{x \to 0}{\lim} \left(\frac{5x}{2\sin 3x} \right) . \left(\frac{\tan 3x}{\sin 3x} \right)$$

$$= \left(\frac{5}{6} \right) . \left(\frac{3}{3} \right) = \frac{5}{6}$$

32. Jika $\displaystyle\lim_{x \to 0} \frac{\sin x}{x} = 1$ maka $\displaystyle\lim_{h \to 0} \frac{\sin(\frac{1}{3}\pi + h) - \sin\frac{1}{3}\pi}{h} = \ldots$

A. $-\frac{1}{2}\sqrt{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{2}\sqrt{2}$

E. $-\frac{1}{2}\sqrt{3}$

Jawaban: C

Gunakan rumus: $\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$

$$\Leftrightarrow \lim_{h \to 0} \frac{\sin(\frac{1}{3}\pi + h) - \sin\frac{1}{3}\pi}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\frac{1}{2}(\frac{2}{3}\pi + h).\sin\frac{1}{2}h}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\frac{1}{3}\pi.\sin\frac{1}{2}h}{h}$$

$$= \lim_{h \to 0} \frac{2\left(\frac{1}{2}\right).\sin\frac{1}{2}h}{h} = \frac{1.\left(\frac{1}{2}\right)}{1} = \frac{1}{2}$$

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$$\Leftrightarrow \lim_{h \to 0} \frac{\cos(\frac{1}{3}\pi + h) - 0}{1} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

33. Nilai dari $\displaystyle\lim_{x \to 0} \frac{\tan 2x.\cos 8x - \tan 2x}{16x^3} = \ldots$

A. -4

B. -6

C. -8

D. -16

E. -32

Jawaban: A

$$\Leftrightarrow \lim_{x \to 0} \frac{\tan 2x.\cos 8x - \tan 2x}{16x^3}$$

$$= \lim_{x \to 0} \frac{\tan 2x.(\cos 8x - 1)}{16x^3}$$

$$= \lim_{x \to 0} \frac{\tan 2x.(-2.\sin^2 4x)}{16x^3}$$

$$= \lim_{x \to 0} (-2).\left(\frac{\tan 2x}{2x}\right).\left(\frac{\sin^2 4x}{8x^2}\right)$$

$$= (-2).(1).\left(\frac{16}{8}\right) = -4$$

34. Jika diketahui $\lim_{x \to 0} \frac{ax\sin x + b}{\cos x - 1} = 1$, maka nilai a dan b yang memenuhi adalah ....

A. $a = \frac{1}{2}, b = 0$
B. $a = 1, b = 1$
C. $a = \frac{1}{2}, b = 0$
D. $a = 1, b = -1$
E. $a = 1, b = 0$

Jawaban: A

$$\Leftrightarrow \lim_{x \to 0} \frac{ax\sin x + b}{\cos x - 1} = 1$$

- untuk $x = 0 \to \frac{0}{0} = \frac{0 + b}{1 - 1} \to b = 0$
- **gunakan L'Hospital :**

$$\lim_{x \to 0} \frac{a.(\sin x) + ax.(\cos x)}{-\sin x}$$

$$\lim_{x \to 0} \left\{ -a - \underbrace{\left(\frac{ax}{\tan x}\right)} \right\}$$

$$-a - a = 1 \to a = -\frac{1}{2}$$

LATIHAN SOAL 19

Pilihlah satu jawaban yang paling tepat pada soal di bawah ini!

SOAL LATIHAN - 1

1. Nilai $\lim\limits_{x \to 4} \dfrac{3x^2 - 14x + 8}{x^2 - 3x - 4} = \ldots$

   A. 4
   B. 2
   C. $\frac{1}{2}$
   D. -2
   E. -4

2. $\text{Limit}_{x \to 1} \dfrac{(3x-1)^2 - 4}{x^2 + 4x - 5} = \ldots$

   A. 0
   B. 1
   C. 2
   D. 4
   E. 8

3. $\text{Limit}_{x \to 2} \left(\dfrac{2x^2 - 8}{x - 2} + \dfrac{x^2 - 2x}{2x - 4} \right) = \ldots$

   A. 5
   B. 6
   C. 8
   D. 9
   E. ∞

4. $\lim\limits_{x \to 3} \dfrac{(x-3)\left(\sqrt{x}+\sqrt{3}\right)}{\sqrt{x}-\sqrt{3}} = ...$

A. 0
B. 3
C. 6
D. 12
E. 15

5. $\lim\limits_{x \to 2} \dfrac{x\sqrt{x}-2\sqrt{x}-2\sqrt{2}+x\sqrt{2}}{\sqrt{x}-\sqrt{2}} = ...$

A. 0
B. 2
C. 4
D. 8
E. 10

6. $\underset{x \to 1}{\text{Limit}} \dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2} = ...$

A. 0
B. $\frac{1}{3}$
C. $\frac{1}{5}$
D. $\frac{1}{7}$
E. $\frac{1}{9}$

7. $\underset{x \to 0}{\text{Limit}} \dfrac{4x}{\sqrt{1+2x}-\sqrt{1-2x}} = ...$

A. 0
B. 1
C. 2
D. 4
E. ∞

8. $\underset{x \to 1}{\text{Limit}} \dfrac{x^{2n}-x}{1-x} = ...$

A. $2n-1$
B. $1-2n$
C. $2n$
D. $2n-2$
E. $2n+2$

9. $\underset{a \to b}{\text{Limit}} \dfrac{a\sqrt{a}-b\sqrt{b}}{\sqrt{a}-\sqrt{b}} = ...$

A. 0
B. 3a
C. $\sqrt[3]{b}$
D. 3b
E. ∞

10. $\underset{x \to 3}{\text{Limit}} \dfrac{1}{(x-3)}\left\{\dfrac{1}{x-7}-\dfrac{2}{x-11}\right\} = ...$

A. $-\frac{1}{24}$
B. $-\frac{1}{32}$
C. 0
D. $\frac{1}{32}$
E. $\frac{1}{24}$

11. Diketahui $\lim\limits_{x \to 5} \dfrac{f(x)g(x)-3g(x)+f(x)-3}{(f(x)-3)(x-5)} = 0$. Nilai $g'(5)$ adalah ….

A. -5
B. -3
C. 0
D. 3
E. 5

12. Jika $\lim\limits_{x \to a}\{f(x)-3g(x)\} = 2$ dan $\lim\limits_{x \to a}\{3f(x)+g(x)\} = 1$, maka $\lim\limits_{x \to a}\{f(x).g(x)\} = ...$

A. $-\frac{1}{2}$
B. $-\frac{1}{4}$
C. $\frac{1}{4}$
D. $\frac{1}{2}$
E. 1

13. Diketahui suku banyak $g(x) = ax^2 + (a-b)x + a$ habis dibagi $(x - 1)$. Jika $\lim\limits_{x \to 1} \dfrac{g(x)}{x^2-2x+1} = \dfrac{1}{3}$, maka nilai $(a + b)$ adalah ….

A. $-\frac{4}{3}$
B. $-\frac{2}{3}$
C. 0
D. $\frac{2}{3}$
E. $\frac{4}{3}$

14. Nilai $\lim\limits_{x\to\infty} \dfrac{3x^4 - 7x^2}{2x^4 + 5x} = ...$

A. $\frac{3}{2}$
B. $\frac{2}{5}$
C. $-\frac{7}{5}$
D. $-\frac{7}{3}$
E. $-\frac{2}{3}$

15. Nilai dari $\lim\limits_{x\to\infty} \dfrac{3x+5}{2x^2+4x+5}$ adalah ....

A. 0
B. $\frac{4}{11}$
C. $\dfrac{3}{4}$
D. 1
E. 6

16. $\operatorname*{Limit}\limits_{x\to\infty} \left(\dfrac{x+3}{2x-1} - \dfrac{2x+5}{x-7}\right) = ...$

A. $-\frac{1}{2}$
B. $-\frac{2}{3}$
C. $-\frac{3}{2}$
D. $-\frac{2}{3}$
E. $-\frac{5}{3}$

17. $\operatorname*{Limit}\limits_{x\to\infty} \dfrac{\sqrt{12x^2 - 4x + 1} + 2\sqrt{x^2}}{x - 2006} = ...$

A. $2\sqrt{3}$
B. $\sqrt{3} + 1$
C. $\sqrt{3} - 1$
D. $2\left(\sqrt{3} - 1\right)$
E. $2\left(\sqrt{3} + 1\right)$

18. $\lim\limits_{x\to\infty} \dfrac{6^x}{2^x + 4^x} = ...$

A. 0
B. 1
C. $\frac{1}{3}$
D. 3
E. ∞

19. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x + 2} - \sqrt{x^2 + 2x + 5} \right) = ...$

A. $\frac{1}{2}$
B. $-\frac{1}{2}$
C. 0
D. -1
E. $-\infty$

20. $\lim\limits_{x \to \infty} \left((3x - 2) - \sqrt{9x^2 - 2x + 5} \right) = ...$

A. 0
B. $-\frac{1}{3}$
C. -1
D. $-\frac{4}{3}$
E. $-\frac{5}{3}$

21. Nilai dari $\lim\limits_{x \to \infty} \left(\sqrt{x^2 - 5x} - x - 2 \right)$ adalah ....

A. ∞
B. $\frac{1}{2}$
C. 0
D. $-\frac{9}{2}$
E. $-\frac{1}{2}$

22. $\lim\limits_{x \to \infty} \left\{ \sqrt{2015x + \sqrt{2015x}} - \sqrt{2015x - \sqrt{2015x}} \right\} = ...$

A. 0
B. 1
C. 2015
D. $\sqrt{2015}$
E. $2\sqrt{2015}$

23. $\lim\limits_{x \to \infty} x \left(\sqrt{25 - \frac{10}{x}} - \sqrt{25 + \frac{10}{x}} \right) = ...$

A. -2
B. -1
C. 0
D. 1
E. ∞

24. $\lim\limits_{x \to \infty} \left(\sqrt[3]{x^3 - 2x^2} - x - 1 \right) = ...$

A. $\frac{5}{3}$
B. $\frac{2}{3}$
C. $\frac{1}{3}$
D. $-\frac{2}{3}$
E. $-\frac{5}{3}$

SOAL LATIHAN - 2

25. $\lim\limits_{x \to 0} \dfrac{3 \sin \frac{1}{2} x}{\tan \frac{1}{3} x} = ...$

A. 0
B. $3\frac{1}{3}$
C. $4\frac{1}{2}$
D. 6
E. 9

26. $\text{Limit}_{x \to 0} \dfrac{\tan 2x . \sin^2 8x}{x^2 \sin 4x} = ...$

A. 32
B. 24
C. 16
D. 8
E. 4

27. $\lim\limits_{x \to 0} \dfrac{x \tan 5x}{\cos 2x \cdot \cos 7x} = ...$

A. $\frac{1}{9}$
B. $-\frac{1}{9}$
C. $\frac{2}{9}$
D. $-\frac{2}{9}$
E. 0

28. $\text{Limit}_{x \to 0} \dfrac{x}{\sin \frac{1}{2} x \cos \frac{1}{2} x} = ...$

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 1
D. 2
E. 4

29. $\underset{x \to \frac{\pi}{2}}{\text{Limit}} \dfrac{\sin(2x-\pi)}{\sqrt{x}-\sqrt{\frac{\pi}{2}}} = \ldots$

A. $\dfrac{1}{\sqrt{2\neq}}$

B. 1

C. 2

D. $\sqrt{2\neq}$

E. $2\sqrt{2\neq}$

30. Jika $\underset{x \to \infty}{\text{limit}}\ x.\sin\left(\dfrac{a}{bx}\right) = b$, a dan b konstanta, maka ....

A. $a = \frac{1}{2}b$

B. $a = b$

C. $a^2 = b$

D. $a = b^2$

E. $a = 2b$

31. $\underset{x \to 0}{\text{Limit}} \dfrac{x^a \sin^4 x}{\sin x^6} = 1$, maka a = ...

A. 1

B. 2

C. 3

D. 4

E. 5

32. $\underset{x \to 1}{\text{Limit}} \dfrac{(x-1)(x-3).\sin(x-1)}{\{(x-1)(x+2)\}^2} = \ldots$

A. $-\frac{2}{9}$

B. $-\frac{2}{3}$

C. 0

D. $\frac{2}{3}$

E. $\frac{4}{9}$

33. $\underset{x \to k}{\text{Limit}} \dfrac{x-k}{\sin(x-k)+2k-2x} = \ldots$

A -1

B. 0

C. $\frac{1}{3}$

D. $\frac{1}{2}$

E. 1

34. Jika diketahui $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$, maka $\lim\limits_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \operatorname{tg} x = ...$

A. 0
B. ∞
C. 1
D. $\frac{1}{2}$
E. 2

35. $\lim\limits_{x \to 0} \frac{\sin(2x^2)}{x^2 + \sin^2 3x} = ...$

A. $\frac{2}{5}$
B. 5
C. $\frac{3}{2}$
D. 0
E. $\frac{1}{5}$

36. $\lim\limits_{x \to 1} \frac{x^2 + x - 2}{\sin\left(2 - \sqrt{x+3}\right)} = ...$

A. −12
B. −6
C. 0
D. 6
E. 12

37. Diketahui $\lim\limits_{y \to 0} \frac{\sin y}{y} = 1$, maka $\lim\limits_{x \to 1} \left(\frac{x^3 - 1}{x - 1} + \frac{\cos(\frac{\pi}{2} - x + 1)}{x - 1} \right) = ...$

A. 0
B. 1
C. 2
D. 3
E. 4

38. $\lim\limits_{x \to 0} \frac{\sin^2 3x . \operatorname{tg} 2x - x^2}{x . \operatorname{tg}^2 3x} = ...$

A. $\frac{23}{9}$
B. $\frac{19}{9}$
C. $\frac{17}{9}$
D. $\frac{8}{9}$
E. 0

39. $\displaystyle \lim_{x \to \frac{\pi}{2}} \frac{\cos^2(\pi - x)}{(2x - \pi).\tan\left(\frac{\pi}{2} - x\right)} = \ldots$

A. -1
B. 1
C. $-\frac{1}{2}$
D. $\frac{1}{2}$
E. 0

40. Jika $\displaystyle \lim_{x \to 0} \frac{\sin x}{x} = 1$ maka $\displaystyle \lim_{x \to 0} \frac{\sin 3x - \sin 3x \cos 2x}{4x^3} = \ldots$

A. $\frac{1}{2}$
B. $\frac{2}{3}$
C. $\frac{3}{4}$
D. $\frac{3}{2}$
E. 3

41. $\displaystyle \lim_{t \to 2} \frac{(t^2 - 5t + 6)\sin(t - 2)}{(t^2 - t - 2)^2} = \ldots$

A. $\frac{1}{3}$
B. $\frac{1}{9}$
C. 0
D. $-\frac{1}{9}$
E. $-\frac{1}{3}$

42. $\displaystyle \lim_{x \to 1} \frac{x^3 - (a+1)x^2 + ax}{(x^2 - a).\text{tg}(x - 1)} = \ldots$

A. 1
B. 1 - a
C. a
D. 0
E. 2 - a

43. Nilai $\displaystyle \lim_{x \to 0} \frac{2x \sin 3x}{1 - \cos 6x} = \ldots$

A. -1
B. $\frac{1}{3}$
C. 0
D. $-\frac{1}{5}$
E. 1

44. $\lim\limits_{x \to 0} \left(\dfrac{1}{x \operatorname{tg} x} - \dfrac{\cos^2 x}{x \sin x} \right) = ...$

A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. 0
D. $-\frac{1}{4}$
E. $-\frac{1}{2}$

45. $\lim\limits_{x \to 1} \dfrac{\sin\left(1-\frac{1}{x}\right) \cos\left(1-\frac{1}{x}\right)}{(x-1)} = ...$

A. -1
B. $-\frac{1}{2}$
C. 0
D. $\frac{1}{2}$
E. 1

46. Jika $\lim\limits_{y \to 0} \dfrac{\sin y}{y} = 1$, maka $\lim\limits_{x \to 1} \dfrac{1-\cos^2(x-1)}{4(x^2-2x+1)} = ...$

A. 0
B. $\frac{1}{4}$
C. $\frac{1}{2}$
D. 1
E. ∞

47. $\lim\limits_{x \to 0} \dfrac{\sqrt{2\sin 2x + 1} - (1 + \sin x)}{x} = ...$

A. $\frac{3}{2}$
B. 1
C. $-\frac{3}{2}$
D. 2
E. 0

48. $\lim\limits_{x \to 0} \dfrac{\sin 2x + \sin 6x + \sin 10x - \sin 18x}{3\sin x - \sin 3x} = ...$

A. 0
B. $\frac{11}{3}$
C. 54
D. 192
E. 212

49. Nilai $\lim\limits_{x \to \frac{\pi}{4}} \sin\left(\frac{\pi}{4} - x\right).\tan\left(x + \frac{\pi}{4}\right)$ adalah ....

A. 2
B. 1
C. 0
D. -1
E. -2

50. $\text{Limit}_{x \to \frac{\pi}{2}} \dfrac{\sqrt{1 - \cos(\pi - 2x)}}{\left|x - \frac{\pi}{2}\right|} = ...$

A. 1
B. -1
C. 0
D. $\sqrt{2}$
E. $\frac{1}{2}\sqrt{2}$