15. (a) (i) If a system is connected by a convolution integral $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$ where $X(t)$ is the input and $Y(t)$ is the output then prove that the system is a linear time invariant system. (8)

(ii) $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{2|\tau|}$. Find μ_y, $S_{YY}(w)$ and $R_{YY}(\tau)$, if the system function is given by $H(w) = \frac{1}{w + i^2}$. (8)

Or

(b) (i) Show that $S_{YY}(w) = |H(w)|^2 S_{XX}(w)$ where $S_{XX}(w)$ and $S_{YY}(w)$ are the power spectral density functions of the input $X(t)$ and the output $Y(t)$ respectively and $H(w)$ is the system transfer function. (8)

(ii) A linear system is described by the impulse response $h(t) = \left(\frac{1}{RC} e^{-t/RC}\right) u(t)$. Assume an input process whose auto correlation function is $A\delta(\tau)$. Find the mean and auto correlation function of the output process. (8)

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Reg. No. :

Question Paper Code : 72070

22/05/17 FN

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fourth Semester

Electronics and Communication Engineering

MA 6451 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that the function $p(x)$ is a legitimate probability mass function of a discrete random variable X, where $p(x) = \begin{cases} \frac{2}{3}\left(\frac{1}{3}\right)^x ; & x = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$

2. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box.

3. Assume that the random variables X and Y have the probability density function $f(x, y)$. What is $E[E(X/Y)]$?

4. If X, Y denote the deviation of variance from the arithmetic mean and if $\rho = 0.5$, $\Sigma XY = 120$; $\sigma_y = 8$; $\Sigma X^2 = 90$, Find n, the number of times.

5. Define wide sense stationary process.

6. If the customers arrive at a bank according to a Poisson process with mean rate 2 per minute, find the probability that during a 1-minute interval no customer arrives.

7. Determine the autocorrelation function of the random process with the spectral density given by $S_{XX}(w) = \begin{cases} S_0 & |w| < w_0 \\ 0 & \text{otherwise} \end{cases}$

8. State any two properties of cross-power density spectrums.

9. Check whether the system $y(t) = x^3(t)$ is linear or not.

10. If $X(t)$ is a WSS process and if $y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)\,du$ then prove that $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A car hire firm has 2 cars. The number of demands for a car on each day is distributed as Poisson variate with mean 0.5. Calculate the proportion of days on which

(1) Neither car is used

(2) Some demand is refused. (8)

(ii) Suppose X has an exponential distribution with mean equal to 10. Determine the value of x such that $P(X < x) = 0.95$. (8)

Or

(b) (i) A random variable Y is defined as $\cos \pi x$, where X has a uniform probability density function over $\left(-\frac{1}{2}, -\frac{1}{2}\right)$. Find the mean and standard deviation. (8)

(ii) A manufacturer produces covers where weight is normal with mean $\mu = 1.950$ g and S.D. $\sigma = 0.025$ g. The covers are sold in lots of 1000. How many covers in a lot may be heavier than 2 g? (8)

12. (a) (i) The joint PDF of (X,Y) is given by $f(x,y) = \begin{cases} 24xy; x>0,\ y>0,\ x+y \le 1 \\ 0 \quad \text{elsewhere} \end{cases}$.

Find the conditional mean and variance of Y given X. (8)

(ii) Two random variables have the joint PDF $f(x,y) = \frac{1}{3}(x+y);\ 0 \le x \le 1,\ 0 \le y \le 2$. Find the correlation coefficient and regression lines. (8)

Or

(b) If X and Y are independent random variables with PDF e^{-x}; $x \ge 0$ and e^{-y}; $y \ge 0$ respectively, find the density function of $U = \frac{X}{X+Y}$ and $V = X+Y$. Check whether U and V are independent random variables. (16)

13. (a) (i) If the random process $\{X(t)\}$ takes the value -1 with probability $\frac{1}{3}$ and takes the value $+1$ with probability $\frac{2}{3}$, find whether $\{X(t)\}$ is a stationary process or not. (8)

(ii) A fisherman catches a fish at a Poisson rate of 2 per hour from a large lake with lots of fish. If he starts fishing at 10.00 a.m. What is the probability that he catches one fish by 10.30 a.m and three fishes by noon? (8)

Or

(b) (i) Using limiting behaviour of homogeneous Markov chain, find steady state probability of the chain given by the transaction probability matrix.

$$P = \begin{pmatrix} 0.1 & 0.6 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$$ (8)

(ii) Prove that random telegraph process $\{y(t)\}$ is a wide sense stationary process. (8)

14. (a) (i) The cross-power spectrum of real random process $\{X(t)\}$ and $\{Y(t)\}$ is given by $S_{XY}(w) = \begin{cases} a + jbw, & |w| < 1 \\ 0 & \text{elsewhere} \end{cases}$. Find the cross correlation function. (8)

(ii) Find the power spectral density of the random process if its autocorrelation function is $R(\tau) = e^{-\alpha\tau^2} \cos w_0 \tau$. (8)

Or

(b) (i) Find the mean, variance and Root-mean square value of the process whose auto correlation function is $R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$. (8)

(ii) Find the autocorrelation of the process $\{X(t)\}$ which the spectral density is given by $S(w) = \begin{cases} 1 + w^2, & |w| \le 1 \\ 0, & |w| > 1. \end{cases}$ (8)